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Goddard Space Flight Center
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ST - AM - 10359

10359

CALCULATION OF THE EFFECT OF CLOSED AIR CIRCULATION ON THE EQUILIBRIUM DISTRIBUTION OF OZONE IN THE EARTH'S ATMOSPHERE

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21 JULY 1965

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ST – AM – 10359

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Geomagnetizm i Aeronomiya
Tom 5, No. 3, 465 – 470,
Izdatel'stvo "NAUKA", 1965.

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SUMMARY

Calculation is made of the settled distribution of ozone density in the atmosphere as a function of the altitude and in the presence of air circulation as a function of the latitude of the spot.

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It is experimentally established that the ozone content in the atmosphere and its distribution in height depend essentially on vertical and horizontal air flows in the atmosphere [1]. Accordingly, the distribution of ozone may serve as an important indicator of atmospheric processes and even of synoptic position. This adds to the investigation of ozone and of its variation at terrestrial globe's scale a direct practical interest.

Theoretically, the influence of purely vertical flows upon the distribution of ozone has been considered in the work [2].

We shall consider in the current work the influence of ozone distribution of a concrete scheme of general atmosphere circulation, including the vertical and horizontal branches of the flow.

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* O RASCHETE VLIYANIYA ZAMKNUTOY VOZDUSHNOY TSIRKULYATSII NA RAVNOVESNOYE RASPREDELENIYE OZONA V ZEMNOY ATMOSFERE

Especially considered here is the closed air circulation [3], of which the vertical and horizontal velocity components V_r and V_φ, taking into account the continuity equation for an exponentially decreasing air density with height, may be written in the form

$$V_r = V_0 \exp\{(\mu - 2(R)(r - R_{\text{Э}})\} \cos b(r - R) \sin aR(\varphi - \varphi_0) / \sin\varphi, \qquad (1)$$

$$V_\varphi = -bV_0 \exp\{(\mu - 2/R)(r - R_{\text{Э}})\} \sin b(r - R) \cos aR(\varphi - \varphi_0) / a \sin\varphi.$$

Here r is the radius-vector directed from the center of the Earth; R is the distance from the center of the Earth to the altitude of the center of the flow region; R_E is the radius of the Earth; φ is the supplement to the latitude; $\varphi_0 = \pi/4$ is the angle of the region's center ; V_0 is a parameter describing the magnitude of the velocity; $\mu = 0.125\,\text{km}^{-1}$ is a quantity, inverse to the "height of uniform atmosphere", characterizing the rate or the rapidity of air density decrease with height [4]; a and b define the dimension of the flow region in such a fashion, that $V_\varphi = 0$ at the northern and southern boundaries of the region and $-V_r = 0$ at the upper and lower ones, that is

$$a = \pi / 2R|\Delta\varphi_{\max}|, \quad b = \pi / 2|\Delta r_{\max}|,$$

where $\Delta\varphi_{\max}$ and $\Delta r_{\max}$ are the maximum deflections of φ and r from the corresponding coordinates of the center of the region.

The flow with velocity components (1) will have the form indicated by arrows in Fig. 1.

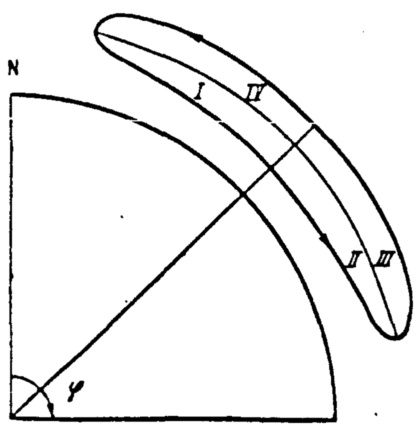

Fig. 1

In this region the continuity equation for ozone density is resolved with the right-hand part, depending on the diffusion and on photochemical processes in the atmosphere, determining the formation and the destruction of ozone

$$\partial\rho / \partial t + \operatorname{div}(\rho \mathbf{V}) = \alpha(\rho_0 - \rho) + D\nabla^2\rho, \qquad (2)$$

where **V** is the flow velocity vector; α is a quantity, inverse to the time of ozone semi-reduction; D is the diffusion coefficient; ρ_0 is the equilibrium density of ozone at $\mathbf{V} = 0$, conditioned only by photochemical processes. It is admitted that

$$\rho_0 = 17(H-10)^3 e^{-0{,}25H} \begin{cases} (1+\sin 2\varphi) & \varphi < \pi/4, \\ [1+(2-\sin 2\varphi)] & \varphi > \pi/4, \end{cases} \qquad (3)$$

where H is the altitude above the sea level.

The distribution of ozone is searched for in a settled air flow, in which $\partial\rho/\partial t = 0$. In the first approximation, considered in the present paper, the influence of diffusion is not taken into account, that is $D = 0$.

Under these assumptions the equation (2) will have the form

$$\frac{\partial(\rho V_r)}{\partial r} + \frac{1}{R}\frac{\partial(\rho V_\varphi)}{\partial\varphi} + \frac{2\rho V_r}{R} + \frac{\rho V_\varphi \operatorname{ctg}\varphi}{R} = \alpha(\rho_0 - \rho). \qquad (4)$$

Here div (ρV) is expressed in spherical coordinates with two variables and not in polar, so as to take into account the simplest way possible the convergence of meridians toward the pole. The unique solution of the equation (4) will be separated, if we start from the condition of ozone density continuity, which is determined by different methods in the four sub-regions of our region, but must coincide on their borders. We shall dwell upon this at further length below.

After substituting in it V_r and V according to the expression (1), the equation (4) will transform into

$$\frac{\partial\rho}{\partial r}\cos b(r-R) - \frac{\partial\rho}{\partial\varphi}\frac{b}{aR}\operatorname{ctg} aR(\varphi-\varphi_0)\sin b(r-R) =$$

$$= -\rho\left[\mu\cos b(r-R) + \frac{\alpha}{V_0}\Phi(r,\varphi)\right] + \frac{\alpha\rho_0}{V_0}\Phi(r,\varphi), \qquad (5)$$

where

$$\Phi(r,\varphi) = \exp\left(\frac{2}{R}-\mu\right)(r-R_3)\frac{\sin\varphi}{\sin aR(\varphi-\varphi_0)}.$$

This linear inhomogenous equation is resolved in partial derivatives of first order by the standard method, with the aid of integration of an auxiliary system of ordinary differential equations [6] :

$$\frac{dr}{\cos b(r-R)} = -\frac{d\varphi}{b\operatorname{ctg} aR(\varphi-\varphi_0)\sin b(r-R)/aR} =$$

$$= \frac{d\rho}{-\rho[\mu\cos b(r-R) + \alpha\Phi(r,\varphi)/V_0] + \alpha\rho_0\Phi(r,\varphi)/V_0}. \qquad (6)$$

The first integral of this system, which is easy to obtain by integrating the first equation of the system, has the form

$$\cos b(r-R)\cos aR(\varphi-\varphi_0)=C. \qquad (7)$$

After that the solution of the equation amounts to integrating the linear ordinary differential equation of the first order

$$\frac{d\rho}{dr}+\left[\mu+\frac{\alpha}{V_0}\frac{\Phi(r,\varphi)}{\cos b(r-R)}\right]\rho=\frac{\alpha\rho_0}{V_0}\frac{\Phi(r,\varphi)}{\cos b(r-R)}, \qquad (8)$$

where φ is expressed through r with the help of (7); this equation (8) may be written in a shorter form as follows

$$d\rho/dr+p(r,c)\rho=f(r,c). \qquad (8a)$$

The dependence of ozone distribution on photochemical processes is manifest, strongest of all, at heights of 20 to 30 km; that is why we may assort a function, describing the true course of α precisely in that interval. On the basis of experimental data and computations by Kondrat'yeva, the following is chosen :

$$\alpha=\alpha_0 e^{0,5H}=6\cdot10^{-13}e^{0,5H}\ sec^{-1}\ {}^{*}$$

The flow region is taken in height from 2.5 to 37.5 km above ground (the center is situated at the height of 20 km) and in latitude, from pole to equator (center at $\varphi=\pi/4$). This is attained by the fact that we assume $b=0.09$, $aR=2$; then $b/a\approx287$.

The solution of the equation (8a) has the form

$$\rho(r,c)=C\exp\left\{-\int p(r,c)\,dr\right\}+\exp\left\{-\int p(r,c)\,dr\right\}\times$$
$$\times\int f(r,c)\exp\left\{\int p(r,c)\,dr\right\}dr. \qquad (9)$$

At $\varphi=\pi/4$, the quantities p(r,c) and f(r,c) pass to infinity; however, it may be shown that in that case the solution will be finite (as should be expected). From the physical standpoint it means that the variation of ozone density in height is not manifest in the solution on the line $\pi/4$, since vertical air flows are absent on that line.

In order to effect the integration in the right-hand part of the equality (9), the trigonometric functions in the expressions p(r,c), f(r,c)

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* Such an expression for α is selected on the basis of calculations of the time of ozone reduction, brought out in the thesis by A.V.Kondrat'yeva, at the Physical Faculty of the Moscow State University (MGU), 1962.

must be approximately represented in the form of polynomials from $x = bh$ ($h = H - 2.5$ km). these polynomials having a different form at $\varphi < \pi/4$ and $\varphi > \pi/4$. At $\varphi > \pi/4$ all the functions will have the index 1, and at $\varphi > \pi/4$ — the index 2.

By the strength of continuity of the solution at $\varphi = \pi/4$, its joining will be achieved, that is, we postulate that $\rho_1(r, \pi/4) = \rho_2(r, \pi/4)$.

It is admitted approximately that

$$\frac{\sin\varphi}{\cos b(r-R)\sin aR(\varphi-\varphi_0)} \approx \begin{cases} [(0{,}05 - 64c^5)x^4 - (0{,}314 - 401{,}92c^5)x^3 + \\ +(0{,}84 - 954{,}56c^5)x^2 - (1{,}088 + 1015{,}84c^5)x + \\ +(0{,}75 - 412{,}584c^5)] \quad \text{at} \quad \varphi < \pi/4, \\ [(0{,}125 + 64c^5)x^4 - (0{,}785 + 401{,}92c^5)x^3 + \\ +(2{,}000 + 954{,}56c^5)x^2 - (2{,}72 + 1015{,}84c^5)x + \\ +(1{,}870 + 412{,}584c^5)] \quad \text{at} \quad \varphi > \pi/4. \end{cases} \qquad (10)$$

The expressions (10) decrease our flow region, for they can be utilized only in specific intervals of H and φ variation. We shall effect computations for the region bounded by 10 and 30 km heights and by 20 and 70° latitudes.

The expressions (10) poorly approximate the initial function at φ close to $\pi/4$. That is why in the band $42{,}5° < \varphi < 47{,}5°$ the calculations will not be conducted and it will be approximately admitted that $\rho_1(x, 42{,}5°) = \rho_2(x, 47{,}5°)$. This is admissible, for in that region the vertical air currents are very weak, while they basically determine the ozone density gradient (as will be seen below).

Utilizing this condition of continuity, we may write the solution, independently from the joining at $\varphi = 42.5 - 47.5°$:

$$\rho_{1,2}(x,\varphi) = \rho^{\circ}_{1,2}(\varphi)\frac{P_{1,2}(x,c)\,|_{x=x_0}}{P_{1,2}(x,c)} - \frac{F_{1,2}(x,c)\,|_{x=x_0}}{P_{1,2}(x,c)} + \frac{F_{1,2}(x,c)}{P_{1,2}(x,c)}, \qquad (11)$$

where

$$P_{i,j}(x,c) = \exp\left\{\int p_{ij}(x,c)\,dx\right\},$$

$$F_{i,j}(x,c) = \int f_{ij}(x,c)\exp\left\{\int p_{ij}(x,c)\,dx\right\}dx,$$

while the solution dependent on the joining is

$$\begin{aligned}\rho_{1,2}(x,\varphi) = \rho^{\circ}_{1,2}(\varphi)&\frac{P_{2,1}(x,c)\,|_{x=x_0}P_{1,2}(x,c)\,|_{\varphi=\varphi_0}}{P_{2,1}(x,c)\,|_{\varphi=\varphi_0}P_{1,2}(x,c)} - \frac{P_{1,2}(x,c)\,|_{\varphi=\varphi_0}F_{2,1}(x,c)\,|_{x=x_0}}{P_{2,1}(x,c)\,|_{\varphi=\varphi_0}P_{1,2}(x,c)} + \\ &\frac{P_{1,2}(x,c)\,|_{\varphi=\varphi_0}F_{2,1}(x,c)\,|_{\varphi=\varphi_0}}{P_{2,1}(x,c)\,|_{\varphi=\varphi_0}P_{1,2}(x,c)} - \frac{F_{1,2}(x,c)\,|_{\varphi=\varphi_0}}{P_{1,2}(x,c)} + \frac{F_{1,2}(x,c)}{P_{1,2}(x,c)},\end{aligned} \qquad (12)$$

where upon integration, c is substituted by its expression in (7). - $\varphi_0 = 42.5$ or 47.5° correspond to the value of x at H = 20 km, $\rho^*(\varphi)$ is the ozone density at H = 20 km.

The functions $p_1(x,c)$, $p_2(x,c)$, $f_1(x,c)$, $f_2(x,c)$ represent the products of the polynomial from x on the exponent. The expressions of the type $\int f_i(x,c)\exp\{\int p_i(x,c)\,dx\}\,dx \quad (i = 1, 2)$ will be integrated by expanding the exponent into series and limiting ourselves to two terms.

The ozone density ρ_1 in the region I (Fig. 1) is determined by the formula (11), the density ρ_2 in the region II — by the formula (12), the density ρ_2 in the region III — by formula (11) and the density ρ_1 in the region IV — by formula (12). At the level $x = x_0$ (H = 20 km) the ozone density may be estimated simultaneously for the regions I and IV, when $\varphi < \varphi_0$ and for the regions II and III when $\varphi > \varphi_0$. By the strength of continuity of the function $\rho(x, \varphi)$ the values of this function at $x = x_0$ are equated at each point, the value of $\rho^*(\varphi)$ – density of ozone at H = = 20 km, being matched according to the obtained sequence of points. Thus, by the strength of the closed condition of current lines, the "boundary condition" is not pre-assigned, but is obtained from the solution itself and has the form

at $V_0 = 10^{-7}$ mk/sec :

$$\rho_1^*(\varphi) = 215(1 + \exp\{-10/|\varphi - 45^\circ|\}),$$
$$\rho_2^*(\varphi) = 215(1 - \exp\{-10/|\varphi - 45^\circ|\}),$$

at $V_0 = 2 \cdot 10^{-7}$ km/sec : (13)

$$\rho_1^*(\varphi) = 215(1 + \exp\{-5/|\varphi - 45^\circ|\}),$$
$$\rho_2^*(\varphi) = 215(1 - \exp\{-12{,}5/|\varphi - 45^\circ|\}).$$

The ozone density was computed at every 5° latitude from 20 to 70° and at every 2 km of altitude from 10 to 30 km. Above 30 km the computation may no longer be conducted in such a fashion, for the restriction of exponent expansion in series to two terms will already be insufficient. Our scheme supposes, that above 37.5 km the air currents are generally absent, and that beginning at about 35 km they are very weak, so that the ozone distribution may be estimated as of this level to be near the photochemical. In the altitude range 30 – 35 km the approximate course of ozone density is interpolated.

The calculations were made for two different velocities V_o, determining the intensity of the whole circulation: 10^{-7} and $2 \cdot 10^{-7}$ km/sec The quantity $V_o = 10^{-7}$ km/sec corresponds to the maximum vertical velocity ~ 0.4 cm/sec and maximum horizontal velocity ~ 1.3 m/sec.
The examples of the results of equilibrium distribution of ozone density obtained from formulas (11) and (12), in the presence of atmosphere circulation at $\varphi = 35, 50,$ (, 70^o, are plotted in Fig. 2.

The curves for the density of ozone are plotted in das es for $V_o = 10^{-7}$ km/sec and by dash-dots for $V_o = 2 \cdot 10^{-7}$ km/sec. The solid line represents the distribution of ρ_o at $V_o = 0$ in accordance with the expression (3).

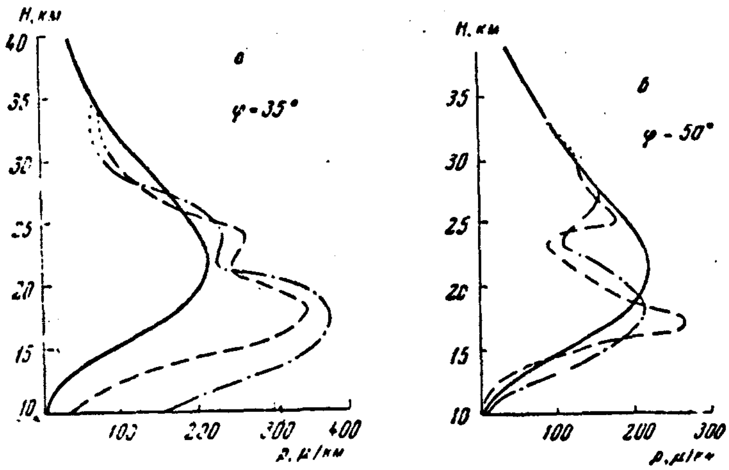

Fig 2 a, b.

It may be seen from these curves, that the indicated atmosphere current scheme giver a distribution of ozone, coinciding on the whole with the experimental data. Thus, in high latitudes, where the descending air currents are strongest, a sharp increase of the total ozone content is observed, alongside with a lowering of the maximum and an increase in ozone concentration in it. As to the lower latitudes, where updrafts are greatest, the total ozone content drops sharply, the maximum is shifted upward, and the concentration of ozone in it decreases. The dual maximum of ozone concentration is also frequently observed. In the given current

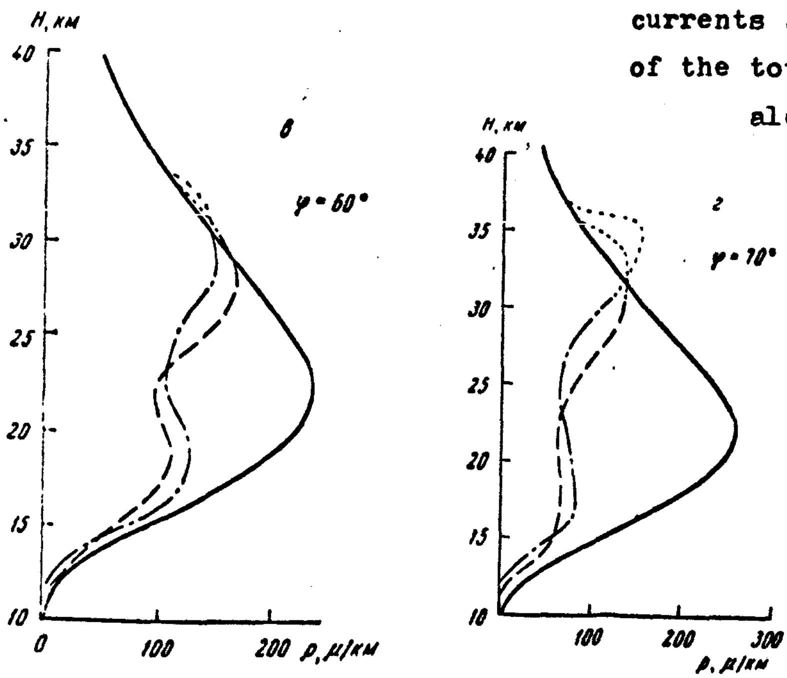

Fig 2 c and d

scheme the upper maximum is forming from the ascending air current, and the lower one — the residual maximum — is conditioned by the maximum that forms from the descending currents at $\varphi < \pi/4$. This maximum is gradually destroyed by the ascending flows, and it vanishes nearly completely in the lowermost latitudes.

It may be seen from these cirves that vertical flows exert a great influence on the distribution of ozone. The greatest variations of ozone content take place precisely where the greatest vertical currents are present. The horizontal currents also affect the distribution of ozone, though to a considerably lesser degree.

It should be noted also, that there exists an interval of air current's vertical velocity, in which the velocity variations are most manifest in the distribution of ozone. Outside the interval of velocity variations in a settled flow, the effect on ozone distribution is immaterial.

The most substantial discrepancy between the results of calculations and observations consists in the sharply overrated ozone content at high latitudes at comparatively low current velocities. This may be partially explained by the fact, that the considered settled flow approaches more or less the settled flow in real atmospheric conditions. This is in part because of the distinction between our scheme and that of the work [3], which consists in that the latter's currents, directed from south to north, spread to heights of 70 — 80 km, whilenin our scheme, the same volume of air is "placed" up to the height of only 37.5 km. That is why the effective air velocity is found to be greater in our scheme. It is nevertheless possible, that the increased ozone reduction in the lower layers, so far disregarded, (for example, aerosol oxydation, etc.), ought to be taken into account.

In conclusion, the author expresses his thanks to A. KH. Khrigan for stating the problem and help in the work.

**** THE END ****

Contract No. NAS-5-3760
Consultants and Designers, Inc.
Arlington, Virginia

Translated by ANDRE L. BRICHANT

on 20 — 21 July 1965

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